GROWTH OF VAPOR LOCKS IN CHANNELS OF SMALL DIAMETERS

V. F. Prisnyakov

UDC 536.423

The results of an analytical investigation into the ejection of liquid from channels by vapor locks formed in these are presented. The theoretical results are compared with experiment.

A number of problems of modern technology require a determination of the flow of liquid boiling in a narrow channel which is shut off at a certain cross section by a cut-off valve (or simply blocked for some reason or other) and connected at the outlet to a receiver of the working substance. The process underlying the formation of vapor locks expelling the liquid from the channel may be arbitrarily divided into two stages. In the first stage we have the ordinary growth of the bubble, up to the instant of complete blocking of the channel, at which it is almost spherical, and in the second stage we have the growth of the vapor bubble in the form of a cylinder with spherical ends.

In the first stage, the usual formula for the growth of vapor bubbles $R = 2c\Phi \mathrm{Ja}\sqrt{a'\tau}$ readily yields the following relationship between the temperature of the vapor in the bubble T, its volume V, and its growth time τ

$$\frac{1}{V^{2/3}}\frac{dV}{d\tau} = \sqrt[3]{36\pi}\, c\Phi \frac{c'\rho'\sqrt{a'}}{r\rho''}\frac{(T'-T)}{\sqrt{\tau}}. \tag{1}$$

The numerical values of the coefficients c and Φ are given in Table 1, which is based on the results of a number of existing investigations.

The pressure change in the bubble p causing the liquid to be expelled from the channel is given [10] by the equation

$$p = p_a - p_z + b_1 G^2 + b_2 \frac{dG}{d\tau}. \tag{2}$$

Here

$$p_z = \rho' gH \sin\alpha;$$

$$b_1 = \frac{1}{2\rho' F^2}\left(\xi_M + \frac{H}{De}\xi\right); \quad b_2 = \frac{H}{F};$$

$$H = \frac{V_0 - V}{F}; \quad V_0 = Fz_s; \quad G = N\rho'\frac{dV}{d\tau} + G_{\mathrm{in}}.$$

Neglecting the change in p (and T), and assuming the outflow of the expelled liquid to be precritical, we obtain the following relationship for the rate of ejection:

$$w = \frac{64}{9} c\Phi^3 a'^{3/2} \mathrm{Ja}^3 \frac{N}{F}\sqrt{\tau}. \tag{3}$$

It should be noted that, according to earlier experiments [11, 12], the ejection of liquid from the channel is mainly determined by the growth of only one vapor lock; when this reaches its exit section, another bubble appears. Hence, if we place the origin of the coordinate system at the point at which the boiling nucleus appears, we may derive an expression for the velocity of the phase boundary determining the

Three-Hundredth Anniversary of the Union of the Ukraine with Russia Dnepropetrovsk State University. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 25, No. 3, pp. 440-444, September, 1973. Original article submitted March 27, 1973.

TABLE 1. Values of the Coefficients c and Φ

Source	1	2,3	4	5	6	7	9
c	$\frac{3}{20}$	$\sqrt{2{,}8+\sqrt{5}}$	$\frac{1}{\sqrt{\pi}}$	$\sqrt{\frac{3}{\pi}}$	$\sqrt{\frac{3}{\pi}}$	$\frac{2}{3\sqrt{\pi}}$	1,94
Φ	$1+\sqrt{1+\frac{400}{3\mathrm{Ja}}}$	$\mathrm{Ja}^{-0,5}$	1	1	$\sqrt{1+\frac{1}{2}\left(\frac{\pi}{6\mathrm{Ja}}\right)^{2/3}+\frac{\pi}{6\mathrm{Ja}}}$	χ	$\frac{\sqrt{\alpha r\rho''}\sqrt[4]{a'\tau}}{\sqrt{\Delta T}\,c'\rho'}$

outflow of liquid from the channel corresponding to the second stage:

$$\frac{dz}{d\tau} = w = c\Phi\,\frac{c'\rho'\sqrt{a'}}{r\rho''}\,\frac{(T'-T)\,S''}{F\sqrt{\tau}},$$

where the total surface area of the vapor lock S" equals $\pi D(z + D)$. Thus finally we obtain the differential equation

$$\frac{dz}{d\tau} = A\,(T'-T)\,\frac{(z+D)}{\sqrt{\tau}}, \tag{4}$$

where

$$A = 4c\Phi\,\frac{c'\rho'\sqrt{a'}}{r\rho'' D}.$$

In order to determine the temperature T we make use of the equation of motion of the ejected liquid (2) in the following form:

$$p = p_a - \rho' g (z_0 - z) + \frac{\xi}{2D}\rho'(z_0 - z)\frac{dz^2}{d\tau} + \rho'(z_0 - z)\frac{d^2z}{d\tau^2},$$

which by virtue of the Clapeyron—Clausius equation may be transformed to the form*

$$T = T_a + T_{\text{av}}\frac{\rho'(z_0 - z)}{r\rho''}\left(g + \frac{\xi}{2D}\left(\frac{dz^2}{d\tau}\right) + \frac{d^2z}{d\tau^2}\right). \tag{5}$$

The system of equations (4)-(5) enables us to derive the law of variation of $z(\tau)$ and $w(\tau) = dz/d\tau$, i.e., $G(\tau)$.

In the case of a constant drop, the flow is determined by an equation giving its lower value. Thus after making the substitution $\bar{H} = z_0 - z/D$ and a number of transformations Eq. (5) gives the following solution

$$G = \rho' F \sqrt{\exp\,(\xi\bar{H})\left[\frac{g}{\xi}\,D\exp(-\xi\bar{H}) - \frac{2\Delta p}{\rho'}\int\exp(-\xi\bar{H})\frac{d\bar{H}}{\bar{H}} + \text{const}\right]} \tag{6'}$$

or after series expansion

$$G = \rho' F \sqrt{\frac{gD}{\xi} - \frac{2\Delta p}{\rho'}\,e^{\xi\bar{H}}\left[\ln\bar{H} - \xi\bar{H} + \left(\frac{\xi}{2}\bar{H}\right)^2 - \left(\frac{\xi\bar{H}}{3}\right)^3 + \text{const}\right]} \tag{6}$$

In the more general case, simple estimates show that in many problems of practical interest the piezometric head and frictional losses have negligibly small effects. Combined consideration of Eqs. (4) and (5) then leads to the equation

$$\frac{dz}{d\tau}\left[\frac{\sqrt{\tau}}{A(z+D)} + \rho'(z_0 - z)\frac{T_{\text{av}}}{r\rho''}\right] = T - T_a. \tag{7}$$

The second term in the brackets is much smaller than the first, and a fairly simple solution to Eq. (7) is therefore

$$\frac{z}{D} = -1 + \exp\,[2A\,(T - T_a)\sqrt{\tau} + \text{const}], \tag{8}$$

where the constant is determined from the condition at $\tau = \tau_1$ (τ_1 is the end of the first stage) $z = D$ const $= -2A(T - T_a)\sqrt{\tau_1}$.

For small channel diameters (small $\bar{z} = z/D$) Eq. (8) may without serious error be transformed into the following (for $\tau_1 \simeq 0$)

$$\frac{z}{D} = \exp\,[2A\,(T - T_a)\sqrt{\tau}]. \tag{9}$$

* For the case of small ΔT. For large temperature drops it is better to use simply the equations for $T = T(p)$; this is less convenient but more accurate.

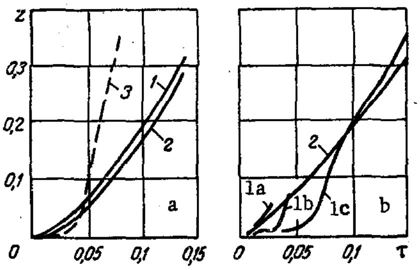

Fig. 1. Change in the length of a vapor lock with time: 1) experiments of [11]; 2) calculations based on Eq. (9); 3) calculations of [11]; 1a), 1b), 1c) growth of individual bubbles a, b, c.

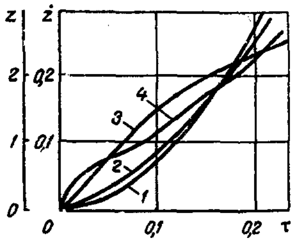

Fig. 2. Change in the length of the lock (1, 2) (for z) and ejection velocity (3, 4) (for ż) of the liquid with time; 1), 3) experiments [11]; 2), 4) calculations based on Eqs. (9) and (4).

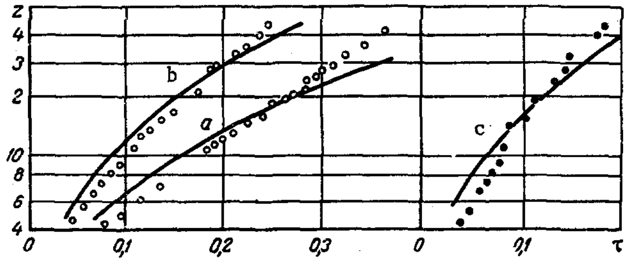

Fig. 3. Comparison between the change in the length of the vapor locks with time as measured in [12] and the calculations based on Eq. (8): a) $\Delta T = 5°$; b) 7°; c) 7.5°C.

We note that the physical parameters in the calculation of A have to be taken at a certain temperature T_{av} between T and T_a.

Any extensive verification of the foregoing theoretical results as to the ejection of liquid from a channel by a sharp pressure surge (with or without heating) is impeded by the limited amount of published experimental data. At the present time we know of only two papers devoted to the experimental study of water boiling in a dead-end glass tube [11,12].

Figure 1a presents the empirical curve (1) of [11] (for T = 122°C) and two calculated curves: 2, based on Eq. (9),* and 3, a numerical computer solution of Eq. (5) obtained by V. Bensimhon [11]. We see that the proposed relationship of Eq. (9) satisfactorily describes the growth of the vapor lock (plunger) in the channel. Figure 1b shows the empirical curves relating to the case in which several vapor bubbles are formed (1a, 1b, 1c at T = 122°C). We see that the lower bubbles show no signs of growth until the upper one has practically escaped. The calculated curve obtained from Eq. (9)† in general closely describes the ejection of the liquid (curve 2). Figure 2 presents the experimental values for the length and growth rate of the vapor locks at T = 110°C. The theoretical values of curve 2 (for z) based on (9) and curve 4 (for $\dot{z} = dz/d\tau$) based on (4) agree with the experimental values (curves 1 and 3 respectively).

A comparison between the calculations and the experiments of [12] is presented in Fig. 3 for superheatings of $\Delta T = 5$; 7; 7.5°C. Since the tube diameter in these experiments was very large, the calculations were based on the more precise equation (8). We see that good agreement between theory and experiment is obtained in this case also, thus indicating the validity of the present results for practical use.

*Calculations for c and Φ were carried out as in [7]; those based on [1] are similar.
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NOTATION

a	is the thermal diffusivity;
c	is the specific heat;
D	is the channel diameter;
G	is the mass flow per second;
$Ja = c'\rho'\Delta T/r\rho''$	is the Jacob number;
F	is the open area;
N	is the number of bubbles generated;
p	is the pressure;
r	is the latent heat of vaporization;
R	is the radius of vapor bubble;
T	is the temperature;
V	is the volume;
w	is the velocity;
z	is the coordinate;
z_s	is the coordinate defining the region of liquid boiling in the channel;
α	is the inclination of the channel axis to the horizontal;
ξ_M and ξ	are the local loss and friction coefficients;
ρ	is the density;
τ	is the time.

Indices

'	refers to the parameters of the liquid;
"	refers to those of the gas;
a	refers to the parameters of the liquid in the receiver;
in	refers to the inlet parameter.

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